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How Membrane Loads Influence the Modal Damping of Flexural Structures

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This paper considers the transverse vibration of flexural structures subjected to preloads that are tangent to the structural midplane; the effects of such membrane loads on the damping of the structural vibration modes are of particular interest. The theory developed indicates that tensile loads increase the natural frequencies of vibration (as is well known), but decrease the modal damping; for beams, the effect on damping is stronger than the effect on frequency. Conversely, compressive loads decrease the natural frequencies and increase the modal damping. These conclusions are consistent for various models of material damping. Available experimental data show good agreement with theory.

Nomenclature

$A_m^*(\omega)$	=	complex modal response, frequency domain
$a_m(t)$	=	modal coordinate
c_{EI}, c_V	=	viscous damping coefficients for: strain-based viscous damping; motion-based viscous damping
EI	=	flexural rigidity or stiffness of beam
F_m	=	harmonic forcing amplitude
L	=	length of beam
m	=	mode number or index
P_{cr}	=	critical buckling load (compressive)
$p_z(x, t)$	=	distributed lateral load acting on beam
Q	=	resonance quality factor $(1/\eta)$
T	=	tension, tensile load
T/P_{cr}	=	nondimensional tension
V	=	potential energy
$w(x, t)$	=	transverse displacement of beam neutral axis
x	=	spatial coordinate that locates cross sections along the neutral axis of a beam
ζ_m	=	modal damping ratio
η_{EI}	=	loss factor of beam in flexure
η_m	=	modal loss factor
η_m/η_{EI}	=	relative modal loss factor with tension
ρA	=	mass per unit length of beam
ω	=	frequency of harmonic forcing
ω_m	=	natural frequency of vibration for mode m
ω_{m0}	=	natural frequency of vibration for mode m without tension
ω_m/ω_0	=	normalized natural frequency of mode m with tension
ω_0	=	natural frequency of vibration for mode 1 (fundamental mode) without tension

I. Introduction

MEMBRANE (or in-plane) loads (or internal prestress) are often encountered in thin-walled aerospace structures. Examples of tensile loads include those associated with spinning helicopter rotor blades, bladed disks in engines, and pressurized aircraft cabins; compressive loads can be due to gravitational forces and acceleration or to pressure loads in buoyant structures. Both tension and compression can be encountered when structures with extended cross sections deform in bending or when temperature or material changes induce internal stresses. The latter situation is experienced in various manufacturing processes involving metallic, composite, and other materials, including silicon microelectromechanical systems (MEMS).

The primary effects of such membrane loads on the linear transverse dynamics of flexural structures (beams, plates, and shells) are generally considered to be changes in the natural frequencies and mode shapes of the structures [1]. These can be qualitatively understood by analogy with the behavior of a string in tension. Such behavior can be exploited in the development of, for example, tunable transducers [2,3].

Other effects of these membrane loads, however, are not as widely appreciated. For instance, such loads can change the damping observed in various modes of structural vibration [4]. This effect can be considerable in applications such as pressurized aircraft fuselages or spinning rotor blades.

A number of researchers report experimental observations of the effects of membrane loads on structural damping, but without a full explanation of the mechanics. The main purpose of this paper is to provide a framework that qualitatively describes these effects and quantitatively captures the details.

Smith and Wereley [5] measured the frequency and damping of the fundamental transverse vibration mode of a spinning rotor blade (beam) in vacuum. Over an angular speed range from 0 to 900 rpm, the resonance frequency increased by 25%, whereas, as shown in Fig. 1, the damping due to a constrained-layer damping treatment decreased by 35%. Smith and Wereley attribute the damping results qualitatively to centrifugal stiffening.

Lesieutre et al. [6] considered the frequency and damping of the vibration modes of a pressurized fuselage sandwich shell. They modified the approach of Baker and Herrmann [7] for the vibration analysis of orthotropic cylindrical sandwich shells to include a complex modulus for the transverse shear stiffness. They found that, for shell properties representative of a contemplated Boeing High Speed Civil Transport (HSCT) aircraft, the damping of some modes of vibration was decreased by as much as 80% by pressurization. Figure 2 shows the modal loss factors as a function of circumferential m and longitudinal n mode numbers. The region of the fuselage most susceptible to this decrease in damping was in the crown above the wings, in which location a thin sandwich panel was designed to carry the predominantly tensile loads associated with pressurization and fuselage bending.

Kosmatka and Mehmed [8] measured the frequency and damping of the fundamental transverse vibration mode of a spinning integrally damped rotor blade (twisted plate) in vacuum. As shown in Fig. 3,

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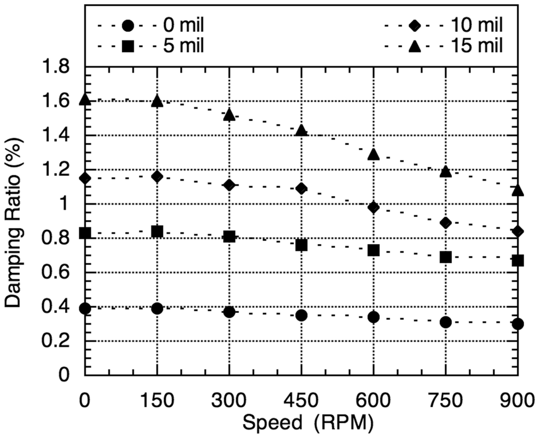

Fig. 1 Damping of the fundamental transverse bending vibration mode of a spinning blade as a function of spin rate [5].

over an angular speed range from 0 to 4000 rpm, the resonance frequency of the first bending mode increased by 300%, whereas the damping decreased by 95%, becoming almost negligible. The resonance frequency of the second bending mode increased by 140%, whereas the damping decreased by 85%. The resonance frequency of the first torsion mode decreased by 15%, whereas the (low) damping was difficult to measure, but perhaps increased modestly. Kosmatka and Mehmed explained the damping results qualitatively in terms of modal strain energy concepts [9], which is consistent with the model proposed herein.

Leland and Wright [10] measured the change in natural frequency and damping of the fundamental transverse vibration mode of a fixed–fixed piezoelectric beam under compressive axial preload. Their interest had to do with vibration-energy-harvesting devices. As shown in Fig. 4, over a range of preloads that decreased the resonance frequency by 25%, the modal damping increased by 80%.

An additional, but related, effect is specialized to piezoelectric structures, in which electrical and mechanical behavior are intimately coupled. In this case, membrane loads are also found to influence the apparent strength of the electromechanical coupling coefficients associated with flexural motion [10,11].

Verbridge et al. [12] reported resonance quality factors Q higher than 200,000 for radio-frequency silicon nitride resonators under tensile stress. Doubly clamped beams made from high-tensile-stress silicon nitride have quality factors higher than those of cantilevered beams and those made from a low-stress material. High-quality factors were thus attributed to high tensile stress. Further research developed a method of controlling the tension in doubly clamped nanomechanical beam resonators. As shown in Fig. 5, they then demonstrated impressive tunability of both frequency and quality factor: as much as several hundred percent for each [3]. Quality factors approaching 400,000 were demonstrated, showing that

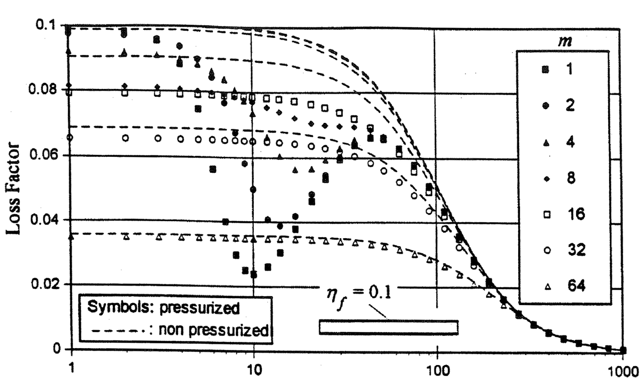

Fig. 2 Total loss factors of a thin sandwich shell, pressurized and nonpressurized, as a function of longitudinal and circumferential mode indices [6]. Loss factor of face sheets $\eta_f = 1$; loss factor of core $\eta_c = 0$.

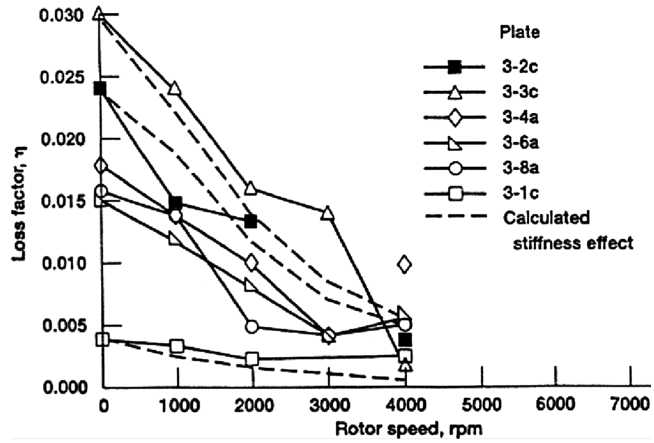

Fig. 3 Variation of the loss factor of the first bending mode with rotor speed for twisted plates [8].

tension can be used as a general material-independent route to increased quality factor. More recent research explored length effects, demonstrating high-aspect-ratio nanoresonators with quality factors exceeding 1,000,000, a value more characteristic of macroscopic oscillators [13]. Material-dependent internal sources of damping are said to provide an upper limit on the frequency-quality factor product fQ. Nevertheless, the precise mechanism by which tension increases the quality factor even in the presence of increased material damping and boundary losses remains unexplained.

II. Model Development

Consider the lateral motion of a beam with internal tension $T(x)$, as shown in Fig. 6. The linear governing equation of motion, neglecting damping, is

$$\rho A(x)\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 w}{\partial x^2}\right) - \frac{\partial}{\partial x}\left(T(x)\frac{\partial w}{\partial x}\right) = p_z(x,t) \quad (1)$$

For convenience, and without loss of generality regarding the effects of tension on damping, specialize to the case of a simply supported uniform beam subjected to a uniform tension load:

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} - T\frac{\partial^2 w}{\partial x} = p_z(x,t) \quad (2)$$

$$w(0,t) = \frac{\partial^2 w(0,t)}{\partial x^2} = 0 \quad \text{and} \quad w(L,t) = \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \quad (3)$$

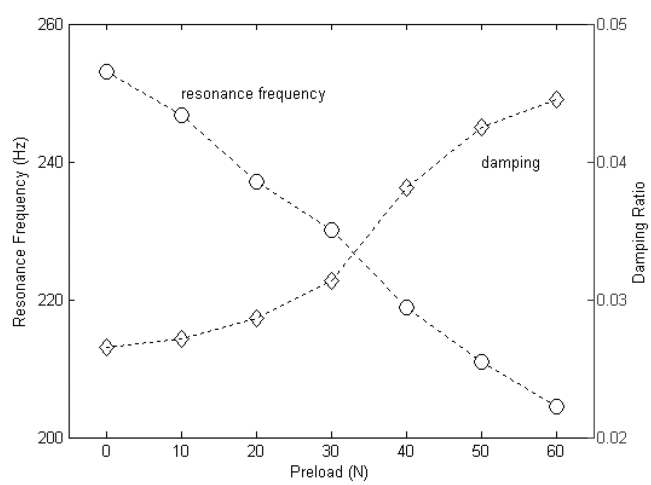

Fig. 4 Natural frequency and damping of the fundamental transverse vibration mode of a fixed–fixed piezoelectric beam subject to compressive axial preload [10].

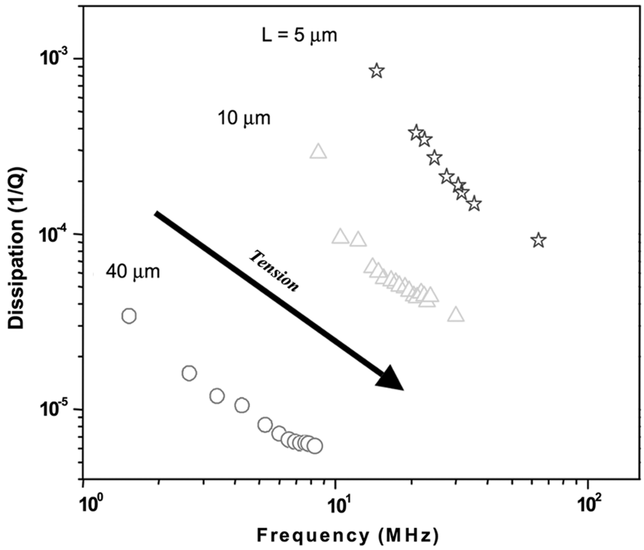

Fig. 5 Modal frequencies and damping for silicon nitride MEMS devices of several lengths, showing the effects of increasing stress [3].

The boundary-value eigenvalue problems involve mode shapes with integer numbers of half-sine waves:

$$w_m(x) = a_m \sin\left(\frac{m\pi x}{L}\right) \qquad m = 1, 2, \ldots \tag{4}$$

The solution of the boundary-value eigenvalue problem for static stability yields an expression for the critical buckling load, shown in Eq. (5). This provides a natural way to nondimensionalize the tension. As will be seen, the fundamental resonance frequency also becomes zero as the compressive load approaches the critical buckling load:

$$P_{\mathrm{cr}} = -T_{\mathrm{cr}} = \frac{\pi^2 EI}{L^2} \qquad (+P \text{ compressive}) \tag{5}$$

The solution of the boundary-value eigenvalue problem for vibration yields an expression for the natural frequencies that shows how they are affected by the tension:

$$\omega_m^2 = \frac{EI(\frac{m\pi}{L})^4 + T(\frac{m\pi}{L})^2}{\rho A} = \left(\frac{m\pi}{L}\right)^4 \left(\frac{EI}{\rho A}\right)\left(1 + \frac{T}{EI(\frac{m\pi}{L})^2}\right) \tag{6a}$$

$$\omega_m = \underbrace{m^2 \underbrace{\left(\pi^2 \sqrt{\frac{EI}{\rho A L^4}}\right)}_{\omega_0}}_{\omega_{m0}} \left(1 + \frac{T}{EI(\frac{m\pi}{L})^2}\right)^{1/2} = \omega_{m0}\left(1 + \frac{T}{m^2 P_{\mathrm{cr}}}\right)^{1/2} \tag{6b}$$

The expression for the natural frequencies [Eq. (6b)] can be viewed as the product of two terms: one representing the nominal natural frequency in the absence of tension and the other representing the effect of tension. As is well known for simply supported beams, the nominal natural frequencies increase with the square of the mode number. As indicated in Eq. (6), the natural frequencies increase with tension, but the effect diminishes with increasing mode number. The higher the tension, the more stringlike the behavior; the lower the tension, the more beamlike.

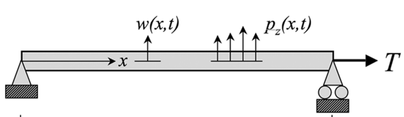

Fig. 6 Simply supported beam with distributed lateral force and tensile axial load.

Next, several damping models are considered: viscous damping (strain-based and motion-based) and hysteretic damping (complex modulus).

None of these models are claimed to accurately capture the physics involved in damping: the viscous models may be said to provide energy dissipation with mathematical simplicity, whereas the complex modulus model does not explicitly exhibit the strong frequency dependence of the viscous models (albeit it at the cost of being a frequency-domain model). In all cases, light damping is assumed, and all vibration modes of interest are underdamped.

The modal damping associated with each of these damping models is addressed. For each mode, the damping in the presence of tension is compared with that of the nominal case with no tension.

A. Viscous Damping

1. Strain-Based Viscous Damping

This involves an internal moment that is proportional to the rate of change of curvature; it could be said to be a dynamic component of the bending moment. After differentiating twice with respect to x to yield the effective distributed lateral force, this adds a term to the equation of motion:

$$\rho A \frac{\partial^2 w}{\partial t^2} + \underbrace{c_{EI}\frac{\partial}{\partial t}\frac{\partial^4 w}{\partial x^4}} + EI\frac{\partial^4 w}{\partial x^4} - T\frac{\partial^2 w}{\partial x} = p_z(x,t) \tag{7}$$

Assuming unforced motion in mode m, the following modal equation of motion is obtained:

$$\ddot{a}_m + \frac{c_{EI}}{\rho A}\left(\frac{m\pi}{L}\right)^4 \dot{a}_m + \frac{1}{\rho A}\left[EI\left(\frac{m\pi}{L}\right)^4 + T\left(\frac{m\pi}{L}\right)^2\right]a_m = 0 \tag{8}$$

By comparing terms with those in the canonical unforced modal equation of motion,

$$\ddot{a}_m + 2\zeta_m\omega_m\dot{a}_m + \omega_m^2 a_m = 0 \tag{9}$$

one can develop an expression for the modal damping ratio ζ_m in terms of the beam properties and the tension.

2. Motion-Based Viscous Damping

This involves an external distributed lateral force that is proportional to the transverse velocity. This adds a term to the equation of motion that is independent of the beam material considered:

$$\rho A \frac{\partial^2 w}{\partial t^2} + \underbrace{c_V \frac{\partial w}{\partial t}}_{\text{motion-based viscous damping}} + EI\frac{\partial^4 w}{\partial x^4} - T\frac{\partial^2 w}{\partial x} = p_z(x,t) \tag{10}$$

$$\ddot{a}_m + \frac{c_V}{\rho A}\dot{a}_m + \frac{1}{\rho A}\left[EI\left(\frac{m\pi}{L}\right)^4 + T\left(\frac{m\pi}{L}\right)^2\right]a_m = 0 \tag{11}$$

Again, one can develop an expression for the modal damping ratio ζ_m in terms of the beam properties and the tension by comparison with the canonical modal equation of motion (9).

B. Hysteretic Damping (Complex Modulus)

Assuming harmonic forcing, the following algebraic equation is obtained in the frequency domain for the response in mode m:

$$\left[-\omega^2\rho A + EI\left(\frac{m\pi}{L}\right)^4 + T\left(\frac{m\pi}{L}\right)^2\right]A_m^*(\omega) = F_m \tag{12}$$

Without forcing, this equation is only satisfied when $\omega = \omega_m$. Next, add a complex term to represent damping in the frequency domain:

$$\left[-\omega^2\rho A + EI(1 + i\underbrace{\eta_{EI}}_{\text{loss factor}})\left(\frac{m\pi}{L}\right)^4 + T\left(\frac{m\pi}{L}\right)^2\right]A_m^*(\omega) = F_m \tag{13}$$

The loss factor in this term is associated with material damping mechanisms and should be considered to vary with frequency. In the presence of damping, harmonic forcing is required to maintain a nonzero (complex) response amplitude. In this case, the equation cannot be satisfied without a forcing term. Nevertheless, comparing terms with the frequency-domain version of the forced canonical modal equation of motion permits one to develop an expression for the modal damping:

$$\left[-\omega^2 + i\eta_{EI}\frac{EI}{\rho A}\left(\frac{m\pi}{L}\right)^4 + \frac{EI}{\rho A}\left(\frac{m\pi}{L}\right)^4 + \frac{T}{\rho A}\left(\frac{m\pi}{L}\right)^2\right]A_m^*(\omega) = F_m \tag{14}$$

$$[-\omega^2 + i\omega 2\zeta_m\omega_m + \omega_m^2]A_m^*(\omega) = F_m \tag{15}$$

An alternate approach involves the use of the so-called modal strain energy (MSE) method [9]; however, this yields identical results when there is a single material modulus.

III. Results

The results for modal damping of a simply supported beam under tension are presented for each of the damping models considered.

A. Viscous Damping

1. Strain-Based Viscous Damping

If the damping mechanism is assumed to be strain-based and viscous in nature, the modal damping ratio in the presence of tension for mode m is found as

$$\zeta_{EI\,m} = \underbrace{\frac{c_{EI}(\frac{m\pi}{L})^2}{2(\rho A\, EI)^{1/2}}}_{\zeta_{EI\,m0}}\frac{1}{(1+\frac{T}{m^2P_{\text{cr}}})^{1/2}} = \frac{\zeta_{EI\,m0}}{(1+\frac{T/P_{\text{cr}}}{m^2})^{1/2}} \tag{16}$$

This can be viewed as the product of two terms: one representing the nominal modal damping ratio in the absence of tension and the other representing the effect of tension. In this case, the nominal modal damping *increases* with the square of the mode number (that is, with the nominal natural frequency). The modal damping decreases with increasing tension (to the one-half power), and the effect diminishes with increasing mode number. In the limit of high tension, the damping for a given mode approaches zero.

2. Motion-Based Viscous Damping

If the damping mechanism is assumed to be motion-based and viscous in nature, the modal damping ratio in the presence of tension for mode m is found as

$$\zeta_{V\,m} = \underbrace{\frac{c_V}{2(\rho A\, EI)^{1/2}(\frac{m\pi}{L})^2}}_{\zeta_{V\,m0}}\frac{1}{(1+\frac{T}{m^2P_{\text{cr}}})^{1/2}} = \frac{\zeta_{V\,m0}}{(1+\frac{T/P_{\text{cr}}}{m^2})^{1/2}} \tag{17}$$

Again, this can be viewed as the product of two terms: one representing the nominal modal damping ratio in the absence of tension and the other representing the effect of tension. In this case, the nominal modal damping *decreases* with the square of the mode number (that is, with the nominal natural frequency). The modal damping decreases with increasing tension (to the one-half power) and approaches zero in the limit of high tension. The effect of tension on modal damping diminishes with increasing mode number.

B. Hysteretic Damping (Complex Modulus)

If the damping mechanism is assumed to be strain-based and hysteretic in nature, modeled using a complex modulus, the modal damping ratio (or modal loss factor) in the presence of tension for mode m is found as

$$\zeta_m = \frac{\eta_{EI}}{2}\frac{1}{(1+\frac{T/P_{\text{cr}}}{m^2})} \quad \text{or } \eta_m = \eta_{EI}\frac{1}{(1+\frac{T/P_{\text{cr}}}{m^2})} \tag{18}$$

Once again, this can be viewed as the product of two terms: one representing the nominal damping in the absence of tension (in this case simply the loss factor of the beam) and the other representing the effect of tension. In this case, the nominal modal loss factor has no explicit dependence on frequency (although it would in practice). The damping for a given mode decreases with increasing tension and approaches zero in the limit of high tension. The effect of tension on damping is strongest for the lowest mode numbers.

In contrast with the viscous damping cases, however, the loss factor decreases with the first power of tension, not the one-half power. As complex-modulus-based damping models have had more success in practice than viscous damping models, Eq. (18) provides the basis for additional discussion.

C. Discussion

For all the damping models considered, increasing tension reduces modal damping relative to its zero-tension value. The effect is strongest for the lowest modes of vibration and increases with increasing tension.

Defining the nondimensional tension as T/P_{cr} and using Eq. (6), define the modal tension stiffness factor as

$$\alpha_m = \frac{\omega_m^2}{\omega_{m0}^2} = \left(1 + \frac{T/P_{\text{cr}}}{m^2}\right) \tag{19a}$$

Then the normalized modal frequency and the relative modal loss factor are defined as

$$\frac{\omega_m}{\omega_0} = \sqrt{\alpha_m}m^2 \tag{19b}$$

$$\frac{\eta_m}{\eta_{EI}} = \frac{1}{\alpha_m} \tag{20}$$

Figure 7 shows the relative modal loss factors and normalized modal frequencies as functions of mode number and nondimensional

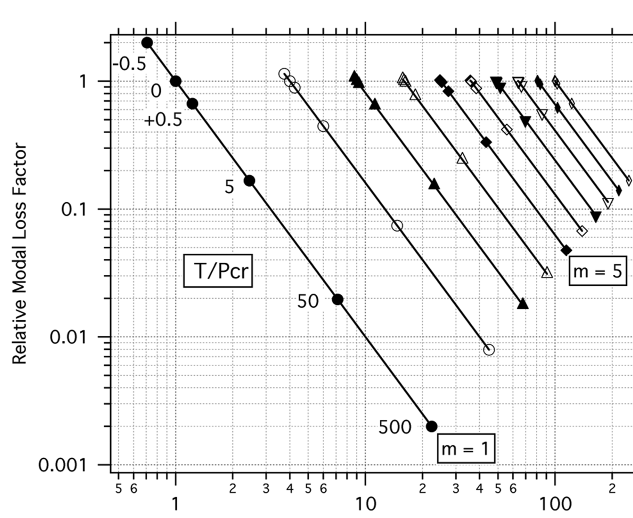

Fig. 7 Relative modal loss factors and normalized modal frequencies of the transverse vibration modes of a beam, as functions of mode number *m* and nondimensional tension T/P_{cr} for a complex modulus model.

tension. The solid lines, one for each mode, show how modal damping and frequency change with tension; discrete values of tension are indicated by the numbers next to markers on the curve for $m = 1$. By visually connecting the markers corresponding to the same value of tension on adjacent curves, one can observe how modal damping and frequency change with mode number. Note that the first point on each curve represents a compressive load; relative to the no-load baseline, modal frequencies decrease, and modal damping increases with compression. Increasing tension clearly reduces modal damping, with the strongest effect on the lowest modes of vibration.

This behavior can be explained physically by considering the membrane load as an effective change in the lateral stiffness of the structure: tensile loads increase it, whereas compressive loads decrease it.

The modal loss factor can be regarded as a weighted sum of the loss factors of the contributors to the potential energy of the system; the weighting factors are the fractions of the potential energy stored in each contributor when the system is in a configuration corresponding to the mode shape of interest:

$$\eta_m = \sum_{\text{parts}} \left(\frac{V_{\text{part}}}{V_{\text{total}}} \right)_m \eta_{\text{part}} \quad \text{with} \quad (V_{\text{total}})_m = \sum_{\text{parts}} (V_{\text{part}})_m \tag{21}$$

The two contributors to the potential energy of the simply supported beam under consideration are the flexural stiffness of the beam and the tension. Realizing that the loss factor associated with the tension is zero, the modal loss factor may be estimated as

$$\eta_m = \left(\frac{V_{EI}}{V_{\text{total}}} \right)_m \eta_{EI} + \left(\frac{V_T}{V_{\text{total}}} \right)_m \eta_T = \left(\frac{V_{EI}}{V_{EI} + V_T} \right)_m \eta_{EI} \tag{22}$$

This is essentially the same result as that obtained in Eq. (18). Compressive loads, by decreasing the effective lateral stiffness, increase the modal damping. If the tension increases, the loss factor decreases, tending to zero in the limit of high tension.

IV. Conclusions

Tensile membrane loads decrease the modal damping of flexural structures, whereas compressive loads increase it; the effect is strongest on the lowest vibration modes. For a complex modulus model, modal damping decreases in direct proportion to the increase in tension, whereas modal frequencies increase in proportion to the square root of the increase in tension. Viscous damping models yield slightly different, but similar, results. This model is qualitatively consistent with a variety of available experimental data.

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